may be integrated[23] and yield

$$F_0(Q,P)=4N^2\left\{\frac{2}{[\alpha^2+(P+Q)^2][\alpha^2+(P-Q)^2]}+\frac{2}{[\beta^2+(P+Q)^2][\beta^2+(P-Q)^2]}-\frac{1}{PQ(\beta^2-\alpha^2)}\times\ln\frac{[\alpha^2+(P+Q)^2][\beta^2+(P-Q)^2]}{[\alpha^2+(P-Q)^2][\beta^2+(P+Q)^2]}\right\},$$

$$G_0(Q,P)=\frac{4N^2-(\beta^2\alpha^2)}{2PQ[\alpha^2+\beta^2+2(P^2+Q^2)]}\times\left\{\frac{1}{\alpha^2+P^2+Q^2}\ln\frac{[\alpha^2+(P+Q)^2]}{[\alpha^2+(P-Q)^2]}-\frac{1}{\beta^2+P^2+Q^2}\ln\frac{[\beta^2+(P+Q)^2]}{[\beta^2+(P-Q)^2]}\right\}.$$

The integration of (14) using the Hulthèn function gives

$$H_2(Q)=\frac{\alpha\beta(\alpha+\beta)}{(\beta-\alpha)^2}\frac{2}{Q}\left[\tan^{-1}\frac{Q}{2\alpha}+\tan^{-1}\frac{Q}{2\beta}-2\tan^{-1}\frac{Q}{\alpha+\beta}\right].$$

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Bound States and Elementary Particles in the Limit $Z_3=0$†

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The limit $Z_3=0$ is studied in the Zachariasen model using dispersion theory techniques. The connection between bound states and elementary particles is demonstrated in this limit and it is shown how Castillejo-Dalitz-Dyson ambiguities are removed.

I. INTRODUCTION

THERE has recently been a great deal of interest in studying field theories in the limit of vanishing renormalization constants.[1–3] Various authors have speculated that in this limit an "elementary particle" can be regarded as a bound state. Vaughn, Aaron, and Amado[4] have discussed the equivalence of the Lee model and potential theory in this limit. Rockmore, and Dowker and Paton[5] considered this problem in the context of the unsubtracted bootstrap model. A convenient model for studying this limit is that proposed by Zachariasen, in which the wave function renormalization can be determined explicitly and is finite. A special case of this theory in which there is no contact interaction has been studied by Acharya[6] and Dowker.[7] Dowker[8] has also discussed a more general case restricting himself to two dimensions and using perturbation theory.

Since the limit $Z_3=0$ is a highly singular one, it is advantageous to have explicit solutions for the quantities of interest and for this reason we shall confine our attention to the Zachariasen model.[9] The comparative simplicity of this theory allows us to see clearly the nature of the difficulties. Our work differs from that of Refs. 6, 7, and 8 in that we consider a wider class of solutions and obtain all our results in terms of finite physical quantities using dispersion theory techniques.

Our results may also be obtained from renormalized perturbation theory although we feel that results stated in terms of unrenormalized coupling constants, masses, etc., tend to be physically misleading.

In Sec. II we present a new dispersion theoretic method for solving the Zachariasen model based on the properties of the vertex function, rather than the denominator function. In Sec. II we exhibit and discuss several apparently different scattering solutions. We also consider the $Z_3=0$ limit of these solutions, and show their equivalence to a bound-state theory. Finally, in an Appendix we discuss a solution which clearly indicates the singular nature of the $Z_3=0$ limit.

II. PROPERTIES OF THE VERTEX AND CALCULATION OF Z_3

The Zachariasen model deals with the interaction of a scalar boson B (with a distinct antiparticle $\bar{B}$) and

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[5] R. M. Rockmore, Phys. Rev. **132**, 878 (1963); J. S. Dowker and J. E. Paton, Nuovo Cimento **30**, 450 (1963).
[6] R. Acharya, Nuovo Cimento **24**, 870 (1962).
[7] J. S. Dowker, Nuovo Cimento **25**, 1135 (1962).
[8] J. S. Dowker, Nuovo Cimento **29**, 551 (1963).
[9] F. Zachariasen, Phys. Rev. **121**, 1851 (1961).

possibly another scalar boson A. We will follow the notation of Ref. 9.

The $B\bar{B}$ scattering amplitude $T(s)$ satisfies the following dispersion relation

$$T(s)=\frac{g^2}{s-\mu^2}-\frac{1}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{|T(s')|^2}{s'-s-i\epsilon}ds', \quad (1)$$

where

$$\rho(s')=\frac{1}{16\pi}\left(\frac{s'-4M^2}{s'}\right)^{1/2}. \quad (2)$$

M and μ are the physical masses of the B and A particles, respectively ($\mu<2M$). If the physical A particle does not exist, then the pole term in (1) is not present.

When the A particle is present, the $\bar{B}BA$ form factor $F(s)$, the $\bar{B}BA$ vertex $\Gamma(s)$, and the A particle propagator $\Delta(s)$[12] are given by

$$F(s)=g-\frac{s-\mu^2}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{T^*(s')F(s')}{(s'-\mu^2)(s'-s-i\epsilon)}ds', \quad (3)$$

$$\Delta(s)=\frac{1}{s-\mu^2}-\frac{1}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{|F(s')|^2}{(s'-\mu^2)^2(s'-s-i\epsilon)}ds', \quad (4)$$

$$\Delta(s)\Gamma(s)=\frac{1}{s-\mu^2}F(s). \quad (5)$$

We now proceed to the calculation of various quantities of physical interest by a method that avoids many of the difficulties due to Castillejo-Dalitz-Dyson (CDD) ambiguities. We shall confine our attention to the case where the physical A particle exists and the form factor has certain asymptotic properties, which we shall specify later.

Our method is based on a consideration of the analytic properties of the vertex function $\Gamma(s)$ given by Eq. (5). From this equation, we see that $\Gamma(s)$ is regular in the cut s plane, and we assume Γ has no poles or zeros. The discontinuity of $\Gamma(s)$ across the cut may be found rather indirectly as follows:

$$\mathrm{Im}\frac{F}{\Gamma}=\mathrm{Im}(s-\mu^2)\Delta(s)=-\frac{\rho(s)|F(s)|^2}{s-\mu^2}. \quad (6)$$

Some trivial manipulation gives

$$\mathrm{Im}\frac{F}{\Gamma}=\frac{\Gamma(s)\,\mathrm{Im}F(s)-F(s)\,\mathrm{Im}\Gamma(s)}{|\Gamma(s)|^2}. \quad (7)$$

Comparing these two and using Eq. (3) gives

$$\mathrm{Im}\frac{1}{\Gamma}=\frac{\rho(s)T^*(s)}{\Gamma^*(s)}-\frac{\rho(s)F^*(s)}{s-\mu^2}. \quad (8)$$

A similar argument shows

$$\begin{aligned}\mathrm{Im}\frac{T}{\Gamma}&=-\frac{\rho(s)|T(s)|^2}{\Gamma^*(s)}+T(s)\,\mathrm{Im}\frac{1}{\Gamma(s)}\\&=\frac{\mathrm{Im}F(s)}{s-\mu^2},\end{aligned} \quad (9)$$

where the last line follows from Eqs. (5) and (8). Hence, we may write a dispersion relation for T/Γ to give

$$\frac{T(s)}{\Gamma(s)}=\frac{g}{s-\mu^2}+a-\frac{(s-\mu^2)}{\pi}\int_{4M^2}^{\infty}\frac{\mathrm{Im}F(s')ds'}{(s'-\mu^2)^2(s'-s-i\epsilon)}, \quad (10)$$

where a is a subtraction constant. Comparing this with the dispersion relation for F [Eq. (3)] we find

$$T/\Gamma=F/(s-\mu^2)+\alpha, \quad (11)$$

where the constant α is given by

$$\alpha=\frac{d}{ds}\left[\frac{(s-\mu^2)T(s)}{\Gamma(s)}-F(s)\right]_{s=\mu^2},$$

and finally

$$T=\Gamma\Delta\Gamma+\alpha\Gamma. \quad (12)$$

We assumed that T/Γ satisfied a once subtracted dispersion relation. It is easy to see that the only effect of further subtractions is to replace α by a polynomial in $(s-\mu^2)$ in Eq. (12). For our present purposes the actual value of α is unimportant. We may remark, however, that the arbitrariness in α corresponds to the CDD ambiguity in the usual N/D method of solving the model.

Substituting this value for T in Eq. (8) and using Eq. (5) gives

$$\mathrm{Im}(1/\Gamma)=\alpha\rho(s). \quad (13)$$

Hence

$$\mathrm{Im}\Gamma=-\alpha\rho(s)|\Gamma(s)|^2, \quad (14)$$

and we can write the two dispersion relations

$$\Gamma(s)=g-\frac{\alpha(s-\mu^2)}{\pi}\int_{4M^2}^{\infty}\frac{\rho(s')|\Gamma(s')|^2}{(s'-s)(s'-\mu^2)}ds', \quad (15)$$

$$\frac{1}{\Gamma(s)}=\frac{1}{g}+\frac{\alpha(s-\mu^2)}{\pi}\int_{4M^2}^{\infty}\frac{\rho(s')ds'}{(s'-\mu^2)(s'-s)}. \quad (16)$$

Now the wave function renormalization constant of particle A is given by[6]

$$Z_3=1-\frac{1}{\pi}\int_{4M^2}^{\infty}\frac{\rho(s')|\Gamma(s')|^2}{(s'-\mu^2)^2}ds'. \quad (17)$$

From Eq. (15) we see that the integral in this expression is equal to

$$-\frac{1}{\alpha}\left(\frac{d\Gamma}{ds}\right)_{s=\mu^2}=\frac{[\Gamma(\mu^2)]^2}{\alpha}\frac{d}{ds}\left(\frac{1}{\Gamma}\right)_{s=\mu^2}=\frac{g^2}{\pi}\int\frac{\rho(s')ds'}{(s'-\mu^2)^2},$$

[10] L. Castillejo, R. H. Dalitz, and F. S. Dyson, Phys. Rev. **101** 453 (1956).
[11] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).
[12] S. D. Drell and F. Zachariasen, Phys. Rev. **119**, 463 (1960).

when the last equality follows from Eq. (16). Hence

$$Z_3=1-\frac{g^2}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{ds'}{(s'-\mu^2)^2} \qquad (18)$$

and is independent of α.

A sufficient condition for the validity of the once subtracted dispersion relations (15) and (16) is that both $|\Gamma(s)|$ and $1/|\Gamma(s)|$ should be bounded by $|s|^{\rho}(\rho<1)$ at infinity. This condition is satisfied by all the solutions we discuss in the next section. A case where it is not satisfied is discussed in the Appendix.

Equation (16) is an explicit expression for $\Gamma(s)$ and we may now calculate all quantities of physical interest merely by evaluating known integrals since[6]

$$[(s-\mu^2)\Delta(s)]^{-1} =1+\frac{s-\mu^2}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{|\Gamma(s')|^2ds'}{(s'-\mu^2)(s'-s-i\epsilon)}. \qquad (19)$$

$F(s)$ is given by (5) and $T(s)$ by (12). It is not necessary to solve any integral equations and no further ambiguities arise.

III. PARTICULAR SOLUTIONS OF THE ZACHARIASEN MODEL

We now proceed to a discussion of particular solutions for the scattering amplitude $T(s)$. It is more convenient for our purpose to use the well known N/D solutions[9,11] of Eq. (1) rather than using Eq. (12) and we shall just point out the connection. For the case when the physical A particle exists two solutions of (1) are given by:

(1) Trilinear theory

$$T_0^{(1)}(s)=\frac{g^2}{(s-\mu^2)D_0^{(1)}(s)}, \qquad (20)$$

where

$$D_0^{(1)}(s)=1+\frac{s-\mu^2}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{g^2}{(s'-\mu^2)^2}\frac{ds'}{s'-s-i\epsilon}. \qquad (21)$$

(2) Combined theory

$$T_0^{(2)}(s)=\left(\lambda+\frac{g^2}{s-\mu^2}\right)\Big/D_0^{(2)}(s), \qquad (22)$$

where

$$D_0^{(2)}(s)=1+\frac{s-\mu^2}{\pi}\int_{4M^2}^{\infty}\rho(s')\left(\frac{g^2}{s'-\mu^2}+\lambda\right) \times\frac{ds'}{(s'-\mu^2)(s'-s-i\epsilon)}. \qquad (23)$$

It is assumed that g^2 and λ are chosen so that the only poles of $T(s)$ are those given by (1).[9] These solutions may be obtained from (12) by choosing $\alpha=0$ and $\alpha=(\lambda/gZ_3)$, respectively.

Let us now consider the possibility of CDD poles in the trilinear theory. That is, we replace $D_0^{(1)}(s)$ by $D^{(1)}(s)$ given by

$$D^{(1)}(s)=D_0^{(1)}(s)+\frac{c(s-\mu^2)}{(s_1-\mu^2)(s_1-s)}, \qquad (24)$$

with $c>0$. Some simple algebra shows that

$$T^{(1)}(s)=\frac{g^2}{(s-\mu^2)D^{(1)}(s)}=\left(\lambda'+\frac{g^2}{s-\mu^2}\right)\Big/\left[1+\frac{s-\mu^2}{\pi}\int_{4M^2}^{\infty}\rho(s')\left(\lambda'+\frac{g^2}{s'-\mu^2}\right)\frac{ds'}{(s'-\mu^2)(s'-s-i\epsilon)}+A(s-\mu^2)\right], \qquad (25)$$

where

$$\lambda'=g^2/(\mu^2-s_1) \qquad (26)$$

$$A=\frac{\lambda'}{g^2}\left(1-\frac{g^2}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{ds'}{(s'-\mu^2)^2}+\frac{\lambda'}{g^2}c\right). \qquad (27)$$

The choice $A\leq 0$, $s_1>\mu^2$ insures that $D^{(1)}(s)$ has no zeros on the physical sheet.

Hence the trilinear theory with a CDD pole may be written in the same form as the combined theory if we choose c such that A vanishes. From (27) and (18) this means

$$c=-g^2Z_3/\lambda'. \qquad (28)$$

In the following we shall assume that (28) is satisfied and shall defer consideration of the general case to the Appendix. By a suitable choice of s_1 it is now always possible to make $T^{(1)}(s)=T_0^{(2)}(s)$ [i.e., $\lambda=\lambda'$].

In order to take the $Z_3=0$ limit of any function we must ensure that all the Z_3 dependence is exhibited explicitly [thus, for example, the $Z_3=0$ limit of Eq. (17) is *not* obtained merely by putting the left-hand side equal to zero since $\Gamma(s)$ itself depends on Z_3]. This condition is satisfied by all the above forms for $T(s)$ and no difficulty arises in taking the limit. Upon doing this [and using (28)] we obtain

$$T_0^{(2)}(s)=T^{(1)}(s)=T_0^{(1)}(s) =1\Big/\left[\frac{s-\mu^2}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{ds'}{(s'-\mu^2)(s'-s-i\epsilon)}\right]. \qquad (29)$$

We should stress the fact that g^2 is no longer arbitrary but is given by

$$\frac{1}{g^2}=\frac{1}{\pi}\int_{4M^2}^{\infty}\frac{\rho(s')ds'}{(s'-\mu^2)^2}. \tag{30}$$

Another solution for $T(s)$ is given by[11]

(3) φ^4 *theory*

$$T_0^{(3)}(s)=\bar{\lambda}/D_0^{(3)}(s)\,, \tag{31}$$

where

$$D_0^{(3)}(s)=1+\frac{s-s_{\bar{\lambda}}}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{\bar{\lambda}}{(s'-s_{\bar{\lambda}})(s'-s-i\epsilon)}ds'\,, \tag{32}$$

where $T_0^{(3)}(s_{\bar{\lambda}})=\bar{\lambda}$ defines the real constant $\bar{\lambda}$. Without any real loss of generality we take $s_{\bar{\lambda}}=0$. $D_0^{(3)}(s)$ has no zeros if $-\frac{1}{2}<\bar{\lambda}<0$, one zero for $0<s<4M^2$ if $\bar{\lambda}<-\frac{1}{2}$ (i.e., a bound state), and a zero for $s<0$ if $\bar{\lambda}>0$ (i.e., a ghost).

Let us first consider $\bar{\lambda}<-\frac{1}{2}$. If (31) is a solution of (1) and if the zero of $D_0^{(3)}(s)$ occurs at μ^2, then we have

$$0=1+\frac{\mu^2}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{\lambda ds'}{s'(s'-\mu^2)}. \tag{33}$$

Defining g^2 by

$$\frac{1}{g^2}=\frac{d}{ds}[T_0^{(3)}(s)]^{-1}\Big|_{s=\mu^2}=\frac{1}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{ds'}{(s'-\mu^2)^2}, \tag{34}$$

some algebra gives

$$T_0^{(3)}=1\bigg/\left[\frac{s-\mu^2}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{ds'}{(s'-\mu^2)(s'-s-i\epsilon)}\right]. \tag{35}$$

Hence, the φ^4 theory with a bound state is identical to the trilinear and combined theories with $Z_3=0$.

If $\bar{\lambda}>-\frac{1}{2}$, we can introduce the A particle as a CDD pole by replacing $D_0^{(3)}(s)$ by $D^{(3)}(s)$ given by

$$D^{(3)}(s)=D_0^{(3)}(s)+\frac{R}{(s-s_3)}\frac{s}{s_3}, \tag{36}$$

where $R>0$. Gell-Mann and Zachariasen have shown that in this case[11]

$$T^{(3)}(s)=\frac{\bar{\lambda}}{D^{(3)}(s)}=\left(\lambda''+\frac{g''^2}{s-\mu''^2}\right)\bigg/\left(1+\frac{s-\mu''^2}{\pi}\int_{4M^2}^{\infty}\rho(s')\left(\lambda''+\frac{g''}{s'-\mu''^2}\right)\times\frac{ds'}{(s'-\mu''^2)(s'-s-i\epsilon)}\right), \tag{37}$$

where μ'', g'', and λ'' are defined by

$$0=1+\frac{\mu''^2}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{\bar{\lambda}}{s'(s'-\mu''^2)}ds'+\frac{R}{(\mu''^2-s_3)}\frac{\mu''^2}{s_3}, \tag{38}$$

$$\frac{1}{g''^2}=\frac{1}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{ds'}{(s'-\mu''^2)^2}-\frac{R}{\bar{\lambda}}\frac{1}{(\mu''^2-s_3)^2}, \tag{39}$$

$$\lambda''=g''^2/(\mu''^2-s_3)\,. \tag{40}$$

Hence, the φ^4 theory with a CDD pole can be cast into the same form as the combined theory. It is *not*, however, always possible to choose the parameters so that the two are numerically identical. If $-\frac{1}{2}<\bar{\lambda}<0$ and $\lambda''<0$, which insures that no ghosts appear, and R and s_3 are chosen so that $\mu''^2=\mu^2$ then it follows from (38) and (39) that

$$\frac{1}{g''^2}-\frac{1}{\pi}\int_{4M^2}^{\infty}\rho(s')\frac{ds'}{(s'-\mu^2)^2}\geq-\frac{D_0^{(3)}(\mu^2)}{\bar{\lambda}\mu^2}>0. \tag{41}$$

Hence, g''^2 may be made equal to g^2 only if g^2 is sufficiently small. This precludes taking the $Z_3=0$ limit [as is obvious from (41)] since g^2 has its maximum value for $Z_3=0$.

We can of course take the limit $s_3\to\infty$ but this is not the same as setting $Z_3=0$, since in order to satisfy (38) we must also let $R\to\infty$ in such a way that R/s_1^2 remains finite. In this limit, Eq. (41) is satisfied with the equality sign holding and we do not get an expression for g^2 in terms of μ^2.

It is possible to obtain a well defined $Z_3=0$ limit of the φ^4 theory with a ghost but this is of little physical interest and we shall not pursue it further.

IV. CONCLUSIONS

We have discussed several solutions of the Zachariasen model and their relationships to each other. In particular we have been able to discuss these relationships in the limit $Z_3=0$. We feel that it is important that this was achieved by using dispersion relations throughout so that no reliance was placed on perturbation theory.

All of the basic solutions for the scattering amplitude $T(s)$ were subject to modification by the usual CDD terms. We have fixed the residue and positions of these poles by requiring that the modified theory can be completely equivalent to the combined theory without a CDD pole. This requirement gave the residue of the pole to be proportional to the wave function renormalization constant of the A particle.

The limit $Z_3=0$ determines the coupling constant g^2 in terms of μ^2. It is worth pointing out that Eq. (18) with $Z_3=0$ is the condition that an unsubtracted dispersion relation hold for $D_0^{(1)}(s)$. Furthermore, in this limit the CDD ambiguity vanishes and there is only one solution of the dispersion relation, namely that given by (29). This solution may be interpreted either as a bound-state theory of the φ^4 type or as the $Z_3=0$ limit of the trilinear *or* combined theories.

Finally, we may point out that all of the results given here agree with renormalized perturbation theory.

There are difficulties in the interpretation of perturbation theory, however, since for consistency it is necessary to assume the bare coupling constants g_0 and λ_0 are always zero.

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APPENDIX

In this Appendix we consider the case of a CDD pole in the pure trilinear theory with a residue that does not satisfy Eq. (28). The scattering amplitude may be written in the form

$$T(s)=\left[\lambda'+\frac{g^2}{s-\mu^2}\right]\Big/D'(s), \tag{A1}$$

with

$$D'(s)=1+(s-\mu^2)\times\int_{4M^2}^{\infty}\rho(s')\left(\lambda'+\frac{g^2}{s'-\mu^2}\right)\frac{ds'}{(s'-\mu^2)(s'-s-i\epsilon)}+A(s-\mu^2).$$

It is convenient to introduce the abbreviation

$$I(s)=\int_{4M^2}^{\infty}\frac{\rho(s')}{(s'-\mu^2)(s'-s-i\epsilon)}ds'. \tag{A2}$$

Then

$$D'(s)=[\lambda'(s-\mu^2)+g^2][I(s)+(1/g^2)-I(\mu^2)]+(\lambda''/g^2)^2C(s-\mu^2). \tag{A3}$$

For (A1) to be a solution of the dispersion relation for T, we require that $D'(s)$ should have no zeros on the physical sheet. The condition for this is

$$0<-\lambda''C/g^2<1-g^2I(\mu^2). \tag{A4}$$

With this form of D', we find $1/\Gamma(s)=0(s)$ as $s\rightarrow\infty$, and the method we used in Sec. II to find Z_3 fails, though a modification of it still works. We may find Z_3 by noting that the solutions given by Zachariasen for the combined theory hold in this case as well, with his $D(s)$ replaced by $D'(s)$. In particular

$$(s-\mu^2)\Delta(s)=1\Big/\left[1+\frac{\lambda''}{g^2}(s-\mu^2)\right]\times\left[\frac{1}{D'(s)}+\frac{\lambda''}{g^2Z_3}(s-\mu^2)\right] \tag{A5}$$

[compare Eq. (45) of Ref. 9].

Hence $\Delta(s)$ will have pole at $s_3=\mu^2-g^2/\lambda''$, unless

$$\frac{1}{D'(s_3)}+\frac{\lambda''}{g^2Z_3}(s_3-\mu^2)=0 \tag{A6}$$

or

$$Z_3=-\lambda''C/g^2.$$

[We may remark in passing that the same method could have been used for the combined theory, and would have given Eq. (18) again.]

Hence for this solution Z_3 is arbitrary apart from the restriction imposed by Eq. (A4), and at first sight the condition $Z_3=0$ gives no restriction on g^2. However, the form of the propagator given in Eq. (A5) is valid only if C does not vanish. If $C=0$, we must go back to the expression for T given by (A1) and (A2), and after a little manipulation we find

$$T(s)=g^2/[(s-\mu^2)D_0^{(3)}(s)], \tag{A7}$$

and hence $Z_3=1-g^2I(\mu^2)$ as before.

It is not known whether the solution with $A\neq 0$ has any simple interpretation in terms of perturbation theory. This example does indicate, however, the highly singular nature of the limit $Z_3\rightarrow 0$.